Cooling Channel Simulation by COSY Infinity

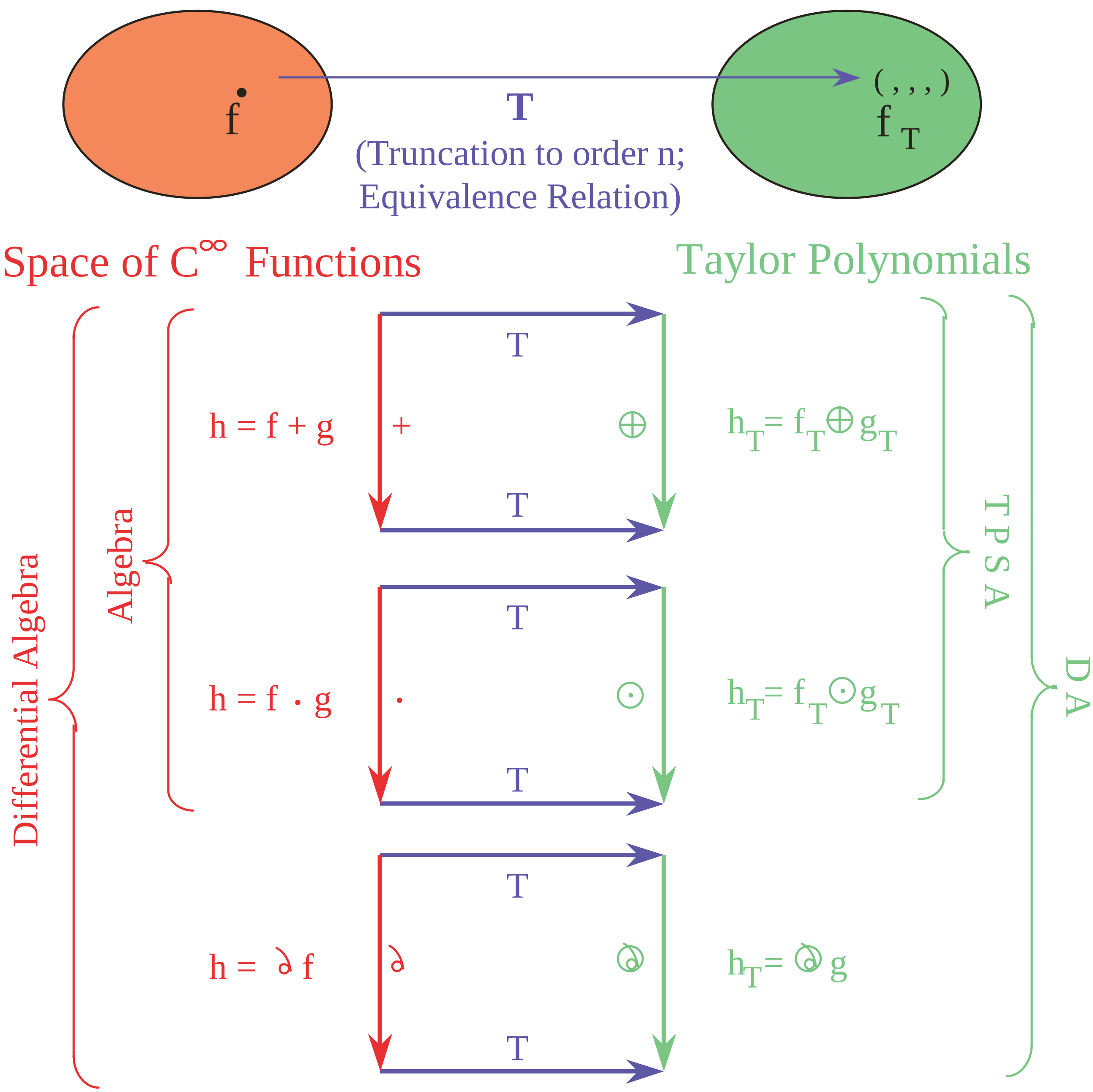
Map Method

- The transfer map is the flow of the system ODE.

$$\overrightarrow{z_f} = \mathcal{M}(\overrightarrow{z_i}, \vec{\delta})$$

- The Differential Algebraic (DA) method allows the computation and manipulation of maps efficiently and elegantly.
- For a repetitive system, only one cell has to be computed. Thus, much faster than tracking codes.
- The Normal Form method can be used for analysis of nonlinear behavior.

Function Algebras



Differential Algebra
(also want “exp”, “sin” etc: Banach DA)

Diagrams commute exactly

Differential Algebra
(even Banach DA)

T: Extracts Information considered relevant

Differential Algebra

The Differential Algebraic method allows a very efficient computation of Taylor transfer maps of beam optical systems.

- works to arbitrary order
- very transparent algorithms;
  effort independent of order
- no restrictions necessary for Hamiltonian, and thus for electric and magnetic fields
- can keep system parameters in map

Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optical systems efficiently.
For example,

DA PDE Solver

- Supply only the midplane field for a midplane symmetric element, the on-axis potential for a rotationally symmetric element.
- The treatment of arbitrary fields is straightforward.
    - o Solenoid fields including the fringe
    - o Magnet (or, Electrostatic) fringe fields (The Enge function fall-off model, or any arbitrary model including the measured data representation.)
    - o Measured fields (Use Gaussian wevelet representation)
    - o Etc.

DA PDE Solver

$$\Delta V = \frac{1}{1+hx}\frac{\partial}{\partial x}\left((1+hx)\frac{\partial V}{\partial x}\right) + \frac{1}{1+hx}\frac{\partial}{\partial s}\left(\frac{1}{1+hx}\frac{\partial V}{\partial s}\right) + \frac{\partial^2 V}{\partial y^2} = 0$$

Rewrite in the form of a Fixed Point Problem.

$$V = V\Big|_{y=0} + \int_y \frac{\partial V}{\partial y}\Bigg|_{y=0}$$

$$-\int_y \int_y \left\{\frac{1}{1+hx}\frac{\partial}{\partial x}\left((1+hx)\frac{\partial V}{\partial x}\right) + \frac{1}{1+hx}\frac{\partial}{\partial s}\left(\frac{1}{1+hx}\frac{\partial V}{\partial s}\right)\right\}$$

COSY code:

```
HF := 1+H*DA(IX) ; HI := 1/HF ; POLD := P ;
LOOP I 2 NOC+2 2 ;
    P := POLD - INTEG(IY,INTEG(IY,
    HI*( DER(IX,HF*DER(IX,P)) + DER(IS,HI*DER(IS,P)) ) ))  ;
    ENDLOOP ;
```